IFSLT 2 Sets

2.1 How many different set are there:
$\{1,2,3,1,3,2,4\}$, $\{1,3,4,\sqrt{4},2\cdot 3\}$,
$\{\frac{9}{3},2,3,4,(-1)^2,3\ +\ 1\}$,$\{1,2,3,4\}$.

2.2 Are the following statements true ?
a) $\{\emptyset,\{\emptyset\}\}\subseteq\{\emptyset,\{\{\emptyset\}\},\{\{\emptyset,\{\emptyset\}\}\}\}$
b) $\{\emptyset,\{\emptyset\}\}\in\{\emptyset,\{\{\emptyset\}\},\{\emptyset,\{\emptyset\}\}\}$
c) $\{\emptyset,\{\emptyset\}\}\subseteq\{\emptyset,\{\{\emptyset\}\},\{\emptyset,\{\emptyset\}\}\}$
d) $\{\emptyset,\{\emptyset\}\}\in\{\emptyset,\{\{\emptyset\}\},\{\{\emptyset,\{\emptyset\}\}\}\}$
e) $\{\emptyset,\{\{\emptyset\}\}\}\in\{\emptyset,\{\{\emptyset\}\},\{\{\emptyset,\{\emptyset\}\}\}\}$

2.3 Let
a) $A=\{\{X\},\emptyset\}$ and
$B=\{\emptyset,\{\{\emptyset\},\{\{\emptyset\}\}\},\{\emptyset,\{\{\emptyset\}\}\},\{\emptyset\}\}$.
b) $A=\{X,\{\emptyset\}\}$ and
$B=\{\{\emptyset,\{\{\emptyset\}\}\},\{\emptyset\},\{\{\{\emptyset\}\},\{\emptyset\}\}\}$.
For what X, holds $A\in B$? For what X holds $A\subseteq B$?

2.4 Give an example of a two-element set such that every its element is its subset.

2.5 Is it true for $a,A,B,\mathcal{A}$?
a) if $a\in A$ and $A\in\mathcal{A}$ then $a\in\mathcal{A}$?
b) if $a\in A$ and $A=B$ then $a\in B$?
c) if $a\in A$ and $A\neq B$ then $a\notin B$?
d) if $a\notin A$ and $A\neq B$ then $a\in B$?

2.6 What can you say about A and B if:
a) $A\cup B=\emptyset$, b) $A-B=\emptyset$, c) $A-B=B-A$.

2.7 Prove or disprove the following equalities? For the false one find simple relations between occurring sets equivalent to the given equality.
a)$A\setminus[(B\setminus C)\cup(C\setminus B)]=A\cap(B\cup -C)\cap(C\cup -B)$,
b)$(A\setminus C)\cup(B\cap C)=[(A\cup C)\cap B]\cup A$,
c) $A\cup(B-C)=[(A\cup B)-C]\cup(A\cap C)$,
d) $(A\cup B\cup C)-(A\cup B)=C$,
e) $(A-B)\cup B=A\cup B$,
f) $A\cap B-A\cap B\cap C=A\cap B-C$,
g) $[(A\cap B)\cup(C\setminus D)]\cap(D\setminus A)=(C\cap D)\setminus(A\cup B)$.

2.8 Let $A\div B=(A-B)\cup(B-A)$. Prove that
a) $A=B\Leftrightarrow A\div B=\emptyset$,
b) $A\div C\subseteq(A\div B)\cup(B\div C)$,
c) $A\div(B\div C)=(A\div B)\div C$,
d) $(A\div B)\cup(A\cap B)=A\cup B$,
e) $C\div(B\setminus A)=(A\cap C)\cup[(B\cup C)\setminus(A\cup(B\cap C))]$
f) $(B\div C)\cap(A\cup B)=[B\div(C\cap A)]\setminus(C\cap B\cap A')$

2.9 Prove or disprove the following equalities?
a) $\mathcal{P}(X\cap Y)=\mathcal{P}(X)\cap\mathcal{P}(Y)$,
b) $\mathcal{P}(X\cup Y)=\mathcal{P}(X)\cup\mathcal{P}(Y)$,
c) $A\times(B\cup C)=(A\times B)\cup(A\times C)$,
d) $A\times(B\cap C)=(A\times B)\cap(A\times C)$,
e) $A\times(B-C)=(A\times B)-(A\times C)$,
f) $(A\times Y)\cap(X\times B)=A\times B$ for $A\subseteq X$, $B\subseteq Y$,
g) $A\times B=B\times A$,
h) $(A\cap B)\times(C\cap D)=(A\cap C)\times(B\cap D)$.

2.10 Let $p(x),q(x)$ be polynomials, let $r(x)=p^2(x)+q^2(x)$. What inclusions holds between the sets of there zeros.